Name _____ School _____ Date _____

Acceleration, Freefall, Symmetry

PURPOSE

- Observe an accelerating object and draw position vs. time (*x-t*), velocity vs. time (*v-t*), and acceleration vs. time (*a-t*) graphs of its motion.
- From an *x-t*, *v-t*, or *a-t* graph of an object with constant acceleration, describe the object's motion.
- From an *x-t*, *v-t*, or *a*-t graph of an object with constant acceleration, draw, within limits, the other two graphs.
- Analyze video frames to determine the acceleration of an object in freefall.
- Investigate symmetries in the motion of an object with constant acceleration

VIRTUAL EQUIPMENT

Virtual dynamics track   VPL Grapher   PENCIL ✎   Frame Flipper tool and video clips (link provided)

DISCUSSION

In our constant velocity lab we used the motion sensor to measure the position of an object 100 times a second and plotted the data as a graph of position vs. time. With our Grapher tool we were able use our (*t*, *x*) data to calculate the rate of change in position, and plot this to create a *v-t* graph. We could then work our way backwards to calculate displacements by calculating the area under the *v-t* graph. The following table summarizes these results as well as some to come in this lab.

Graph	Slope	Area
Position, time, *x-t*	Velocity	--------------
Velocity, time, *v-t*		Displacement, Δ*x*
Acceleration, time, *a-t*	jerk, jolt, surge, lurch	Change in velocity, Δ*v*

Table 1 – The Meanings of the Slopes and Areas of *x-t*, *v-t*, and *a-t* graphs

We've also learned that

1. a straight-line position, time graph indicates a _____ velocity.

2. a straight-line, horizontal position, time graph indicates _____ velocity.

3. a steep, position, time graph indicates a relatively _____ velocity.

We now turn our attention to **acceleration**. We'll look at position, velocity, and acceleration graphs. The table above reminds us that the slope of the position, time graph is the velocity. The slope of the acceleration, time graph and the area under a position, time graph are not of interest to us in this course.

4. In the table above fill in the name that we give to the slope of the velocity, time graph.

You now have a complete list of the two slopes and two areas that you need to know about.

PROCEDURE

I. Motion on an Incline

A. Position on an incline

Start up the Dynamics Track apparatus. **Don't turn on the motion sensor until you're instructed to.**

We want to observe the motion of the cart moving freely up the ramp and then back down.

The low-friction track has a feature that you may not have noticed. If you move your mouse over the small orange rectangle near either end of the track your pointer changes and the message box informs you that you can tilt the track by dragging up or down. Give it a try. It's also important to realize that the **x-axis rotates with the track**.

Setting the track angle can be tricky. Try to set the angle at 3.1°, then 3.2°, then 3.3°. It's not so easy. Try this. Click on the left handle and with the button still down drag toward the center of the track. Try setting the angle to 3.2° now. It's nearly impossible. Now try the same but drag away from the center of the track. Much easier. Think about how that might work.
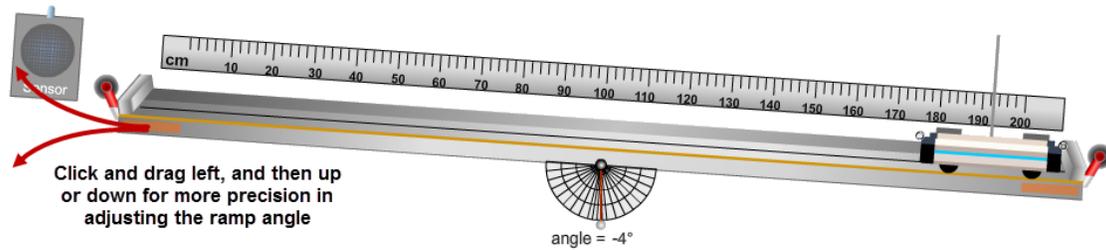

Click and drag left, and then up or down for more precision in adjusting the ramp angle

angle = -4°

Figure 1 – Dynamics Track

1. Adjust your track to an angle between -2° and -6°. Your chosen track angle = – _____ °

Set Recoil to 0. Recoil (×V$_o$) 0   Turn on the ruler.

Use the "V$_o$" and "Go→" tools with a negative velocity to find an initial velocity that will send the cart to about the 20-cm point before returning back to the bottom. Remember, the cart's location is defined as the location of its mast.

Go → Vo -160 cm/s
(Sample negative v$_o$)
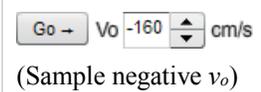
2. Your chosen initial velocity, v_o = – _____ cm/s

Your goal in this lab is to learn to observe an accelerated motion and draw the x-t, v-t, and a-t graphs describing that motion. The three graphs won't look at all alike. You might say they use the same vocabulary but the words have different meanings. The slope of an x-t graph means something entirely different than the slope of a v-t or a-t graph. You'll be asked several times to draw your prediction of what a given graph will look like. Then you'll use the apparatus or Grapher to check your work. It doesn't matter so much if your prediction is correct or incorrect. What's important is that you have a clearly reasoned prediction that you can check against your experimental results. That's where the learning happens.

Launch your cart a few times (no sensor!) and think about what you'd expect for the shape of an x-t graph of this motion – **starting at the bottom, up to near the top, and then back to the bottom of the ramp**.

Let's do some hand drawing on Graph IA1a. You'll (later) start the sensor, and hence the graph, and then wait about two seconds <u>before</u> launching the cart. So the x-t graph will indicate a stationary cart for maybe 2 seconds. From your velocity graphing lab you know that this would produce a horizontal line on an x-t graph. But where? The cart is initially at its lowest point. Does that correspond to a low position value on the graph, like 10 cm? Look at the ruler.

3. Approximately what is at the starting position, x$_o$, of the cart? _____ m

Draw a dot in Figure IA1a at approximately (t_o = 0 s, x_o) showing where the cart is initially. Draw another dot at (t = 2 s, x_o), just before launch. Draw another pair of dots for the at rest period after it returns to the bottom, maybe at eight and 10 seconds. Draw a horizontal line between the first two dots and another between the second two dots. OK, the accelerated motion that we're interested in is just from the "2-second" point to the "8-second" point. So erase the two lines and first and last dots. They were used just to get you oriented on the graph. Also, the exact times of 2 and 8 seconds are not important. They depend on your track angle and initial velocity. Right now we're only interested in the shape of the graph.

The cart will only be at the top of the track <u>at</u> an instant, say, at t_{top} = 5 s. Launch the cart a few times to get a reasonable value for the position, x_{top}, when the cart is at the top of the track.

4. Approximately what is x_{top}? _____ m   Put one dot at (t_{top} = 5 s, x_{top}).

5. Drawing lightly with your pencil, connect the points at 2 and 5 seconds with a straight line. Repeat for the points at 5 and 8 seconds. You should now have a nice, symmetrical, V. The slopes of the two lines would indicate two periods of constant velocity. Is that correct? No, the speed actually smoothly changes over time so the slope should also smoothly change. It should be steep when the speed is large, and horizontal when the cart is not moving. And it should never be vertical. What about the signs of the slopes? You learned about that in the velocity lab too. Add a smooth curve whose slope changes to match this pattern. Erase the V.

6. Describe the motion of the cart <u>and</u> the related shape your graph. You should use words like rise, fall, slow, fast, slowing down, speeding up, positive direction, negative direction, steep, ± slope.

   a. For the trip up the ramp,

   _____

   _____

   b. When the cart is <u>at</u> the top of the ramp,

   _____

   _____

   c. For the trip back down the ramp,

   _____

   _____

7. Use the Sketch tool to create a screen-sketch of your prediction. You'll need to move the time axis to the bottom and make use of the Sketch curve tool. ⌂ Take a Screenshot, save it as "Accel_IA1a.png" and print out your figure. Attach it in the space provided.

8. Now launch the cart with the Sensor on. Sweet! Draw the observed *x-t* graph in Graph IA1b. (Omit the constant velocity periods.)

9. Copy your data to Grapher as follows.
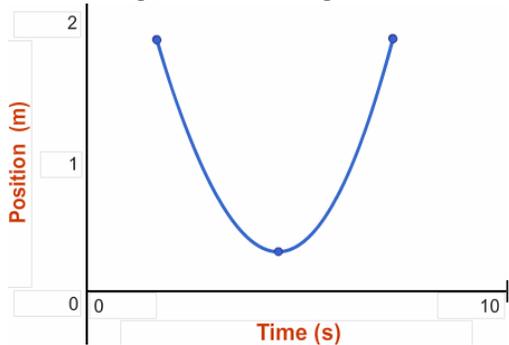   a. In the Dynamics Track apparatus, click "Copy Data to Clipboard."
   b. Open Grapher. The Graph #1 check box should be on by default. Don't turn on any others yet.
       Click in the box below and hit Ctrl+V to paste data.
   c. Paste your data into Grapher using the Ctrl-V tool. ☐
   d. For the graph title use – "Position vs. time"

IA1a. Prediction: Up, then down ramp ✏️ [Sketch] 📷   IA1b. Observation: Up, then down ramp ✏️
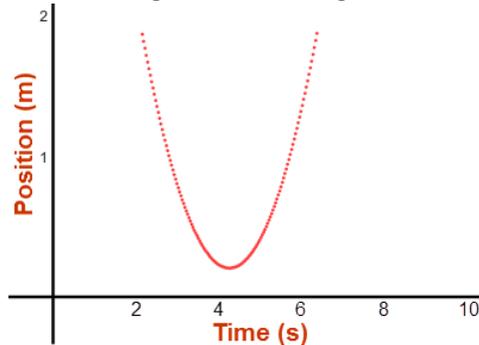
Before we move on, note the obvious symmetry in the cart's motion. The trip up the ramp is replayed in reverse on the way back down. The graph is also symmetrical about a line passing through the maximum position. We'll return to this shortly.
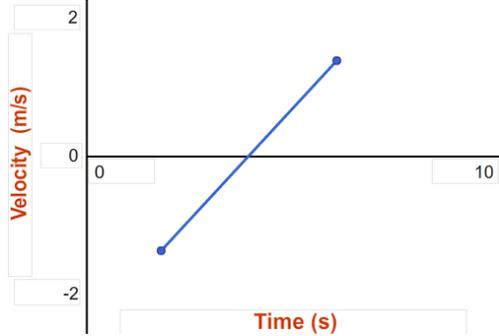
B. Velocity on an incline

We now want to consider a graph of velocity vs. time for the cart's motion up and back down the track. What would that look like? As usual, a concise conversation with your lab partner would be a good idea.

1. The cart starts off traveling in a negative direction. Describe the cart's motion in terms of its changing velocity. Be sure to include the **direction of the velocity** which at a given instant will be **negative, positive, or neither**. Also include the **magnitude**, which will be **large** when the cart is going fast, **small** when it's going slowly, and **zero** when it's at rest.

   _____

   _____

2. Based on your description, draw your prediction of the shape of the velocity, time graph you'd expect in Graph IB1a. Note: Take some time to think about this. Most students miss this one entirely. It might be good to consider the cart's velocity at several instants and plot points on the graph to match. Then think about the shape that would connect them.

IB1a. Prediction: Up, then down ramp ✏️ Sketch 📷   IB1b. Observation: Up, then down ramp ✏️
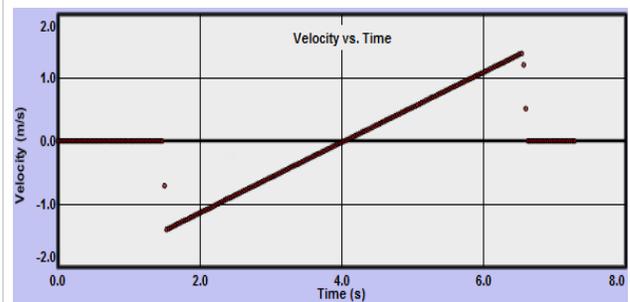
3. Use Sketch to create a sketch of your prediction. Take a Screenshot, save it as "Accel_IB1b.png" and print out your figure. Attach it in the space provided.

4. In Grapher add graph #2 and click the Graphs button to choose a Y' ("Y prime") graph. Y' is an abbreviation for the derivative of value on the y-axis with respect to the value on the x-axis. So in this case it's the derivative of the position, x with respect to time, t which is the slope of the x-t graph, and thus the cart's velocity.

5. You now have a v-t graph. Add an appropriate title.

    You may need to adjust the maximum and minimum values of the velocity axis so that the interval where it was freely moving almost fills the graph vertically. (E.g., -2 m/s to +2 m/s) To do this, click in the v-t graph to select it. Then right-click and select "Manually Resize Axes." The settings shown in the figure to the right should work pretty well.

    Draw your observed graph in Graph IB1b.

6. Any surprises? It seems that the v-t graph should change when the cart gets to the top of the ramp, but instead it just keeps on rising. What about the v-t graph *does* change when the cart reaches the top?

Look at your graph and your answer to #1 in this section. It's very important that your words say the same thing as the graph. It's also important that you convince yourself that the graph makes sense. Feel free to change your answer to #1.

7. Clearly describe your v-t graph. **Not the motion!** You should know the details we're looking for now.

This mangled bit of literature may help you prepare for the next question:

>   *Once upon a time a moocow went down the road <u>for</u> four hours. It*
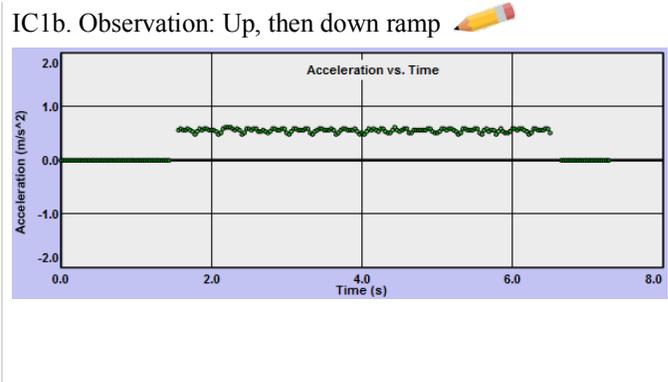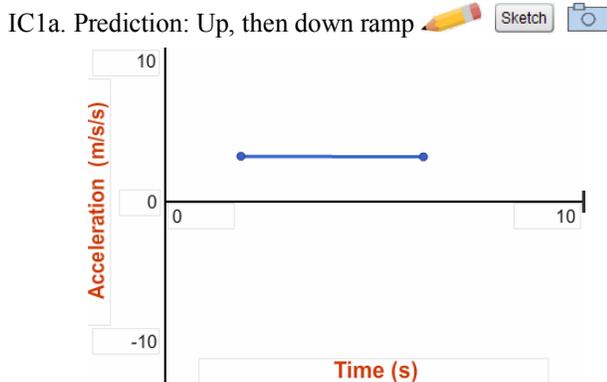>   *arrived at the home of baby tuckoo <u>at</u> three o'clock in the afternoon.*

8. Consider the cart at the highest point it reaches along the track. Its velocity there is zero.
    Does it have this velocity of zero **for an instant** or **at an instant** at the top of its motion? Choose one of these two and explain your answer. (Look at your graph!)

_____

_____

C. Acceleration on an incline:

1. Look back at Table 1. How do we find acceleration from a velocity, time graph?

_____

2. Based on that answer you should be able to look at your *v-t* graph and predict the shape of the corresponding *a-t* graph.

    a. Draw your prediction of the shape of the cart's *a-t* graph in Graph IC1a.
    b. Use Sketch to create a sketch of your prediction.
    c. Take a Screenshot, save it as "Accel_IC1a.png", and print it out. Attach it in the space provided.

Now use Grapher to create the acceleration, time graph. It will do this by taking the derivative of the velocity with respect to time. Remember, we found that the term derivative refers the rate of change of some quantity. The derivative of the position with respect to the time is the velocity. The derivative of the velocity with respect to time is acceleration. That makes it the second derivative of position with respect to time. You'll need to add Graph #3 and set it to Y" to display the acceleration. Manually change the graph scale to just display *y* between -2 and +2.

3. Draw Grapher's graph in Graph IC1b.

IC1a. Prediction: Up, then down ramp ✏️ [Sketch] 📷

IC1b. Observation: Up, then down ramp ✏️

4. Take a Screenshot of all three graphs using Screenshot in Grapher. Save it as "Accel_rampGraphs.png" and print it out to record the relationships among the three graphs. Attach it at the end of your lab. 📷

You may have been surprised by what you saw. Shouldn't the acceleration be somehow different going up than going down, just like we saw with velocity? This is what was meant by the earlier remark – "You might say they use the same vocabulary but the words have different meanings." The cart's *constantly* changing velocity is indicated by a *constantly changing slope* on an *x-t* graph, a *constant slope* on a *v-t* graph and a *constant value* on an *a-t* graph!

Let's look at one other tricky, but important concept – **the meaning of the sign of the acceleration**.

For one-dimensional motion along a line (axis) we choose an arbitrary point where position is defined to be zero. Positions on one side of that point have negative values. Those on the other side have positive values.

- The sign of the position indicates the location, on one side or the other, of an object relative to the zero point.
- The sign of the velocity indicates the direction of motion toward the positive or negative end of the axis.
- The sign of the acceleration indicates the direction of the change in velocity, Δv.

Position and velocity are not so bad. But let's look at the acceleration. Suppose your cart was launched at -2.0 m/s, and because of the slope of the track it had an acceleration of +1.0 m/s^2. Its velocity changes (Δv) by +1.0 m/s every second. Using $\Delta v = a \, \Delta t$, we can calculate what happens during each successive second.

- During the 1st second it changes **from**   -2.0 m/s, **by +1.0 m/s**, **to** -1.0 m/s.   It's slowing down in the– direction.
- During the 2nd second it changes **from**   -1.0 m/s, **by +1.0 m/s**, **to** 0.0 m/s.   It's slowing down in the– direction.
- During the 3rd second it changes **from**   0.0 m/s, **by +1.0 m/s**, **to** +1.0 m/s.   It's speeding up in the + direction.
- During the 4th second it changes **from** +1.0 m/s, **by +1.0 m/s**, **to** +2.0 m/s.   It's speeding up in the + direction.

A constant velocity means that something changes its position at a constant rate.

A constant acceleration means that something changes its velocity at a constant rate.

And if that acceleration is in the opposite direction of its motion, it can not only slow down, but change direction.

Here is a pair of statements that are helpful in understanding the sign of the acceleration:

- When an object is speeding up, its velocity and acceleration have the same signs.
- When an object is slowing down, its velocity and acceleration have opposite signs.

The Dynamic Vectors tool is very useful for illustrating this relationship. Click on the multi-color vector icon.
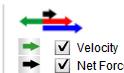
Turn off the ruler. On the left side of the screen uncheck all but the Velocity and Net Force check boxes.

Launch the cart with the same ramp angle and initial velocity that you've been using throughout the lab. The green vector arrow represents the velocity. Notice how it continues to change in magnitude and eventually direction, starting from full length going up the ramp to zero length and then pointing down the ramp and increasing in length.

It's a good exercise to think about how each graph, *x-t*, *v-t*, *a-t*, predicts how this vector changes in magnitude and direction.

The black arrow represents both the total force on the cart and its acceleration. We'll discuss these forces soon but for now, just notice the acceleration. It's constant and always directed down the ramp. Now look at the two bulleted statements above. Hopefully you can "see" what they're saying now.

5. When the cart is rolling up the ramp, its velocity is negative since it's moving in the negative direction. And the cart is slowing down.	The sign of its acceleration is _____
6. When the cart is rolling down the ramp, its velocity is positive since it's moving in the positive direction. And it's speeding up.	The sign of its acceleration is _____

You now have two graphs that can provide values for the acceleration of the cart. The acceleration, time graph has a varying height that clearly varies around a certain average value during the period of constant acceleration. Select this part of the graph by dragging across it. Check the Stats checkbox to find the average (mean) value of the acceleration. Record it below.

Another way of determining the acceleration is from the slope of the line of best fit of the velocity graph. Drag across the velocity graph where the acceleration (slope) is constant and click the Linear Fit checkbox. The slope is the acceleration. Record your results below.

7. Acceleration from acceleration, time graph _____ m/s^2

8. Acceleration from velocity, time graph _____ m/s^2

D. Find the acceleration due to gravity, g from the acceleration of our cart on an incline

When Galileo studied acceleration along a ramp he saw that the acceleration increased with the angle of the ramp. He wasn't able to test his measurements given the limitations of using a water clock to measure time, but he did predict that some maximum acceleration would be reached when the ramp was tilted to 90°. We refer to this vertical acceleration of a freely falling object as the **acceleration due to gravity, g**. Later in the course we'll show that g should equal a/sin(θ) where a is the acceleration for some ramp angle, θ.

Let's see what your results would predict for g. Determine this value using the average of your two acceleration values.

1. Predicted maximum acceleration, $a_{avg}/sin(\theta) =$ _____ m/s^2

Show calculations here.

Compare this result to the value assumed in the lab, 9.80 m/s² by finding the percentage error.

2.   Percentage error_____ %

Show calculations here.

E. The area under an acceleration vs. time graph

We've now thoroughly studied everything in the table on page one except the areas under an *a-t* graph. The height of the graph is acceleration, the width is Δt. The area is the product of these two. This is best understood from the definition of acceleration.

$$a = \frac{(v_f - v_o)}{\Delta t}$$

Rearranging we get $v_f - v_o = a\,\Delta t$

So the area, $a\,\Delta t$ is just the change in velocity over the interval Δt. Let's find Δv from $v_f - v_o$ and from $a\,\Delta t$ for the full trip up and back and compare. Let's find everything from our graphs.

First the *a-t* graph. Click in that graph.

Turn on the Examine tool and move your cursor over the left-most point on the part of the *a-t* graph when the cart was actually in motion – the plateau region. (You can move your cursor anywhere above or below this point.)

1.   In the Examine box you'll see the time and acceleration (*t, a*) for that point. Doing the same with the right-most point you'll find your final time and acceleration for the trip. Record these values.

$t_o, a_o =$ _____ s,   _____ m/s²

$t_f, a_f =$ _____ s,   _____ m/s²

We could calculate $a\,\Delta t$ from data this but the acceleration actually varies a lot. A better way of getting $a\,\Delta t$ is to find the area under the *a-t* graph.

We'll use the Integrate tool which calculates the area under the graph taking into account all the variation. Click its check box to turn it on. You want to click and drag across the *a-t* graph from t_o to t_f. Zooming helps, so right-click somewhere near the center of the *a-t* graph. Click "Zoom in" once. As you move your pointer a vertical line will move with it. When the examine circle encircles the first point on the plateau, click and start to drag right. When it encircles the last point on the plateau, release the mouse. Right-click and turn off Zoom.

The area will be listed in the integrate box. Its units are m/s. Turn of integrate once you get your result.

2. Δv = area under *a-t* graph = _____ m/s

3.   Now read v_o and v_f from the *v-t* graph using the Examine tool. One problem. It has one extra data point on each end due to the way that the acceleration values are calculated. You've recorded t_o, and t_f above so you can use this to check that you're working with the right points.

$t_o, v_o =$ _____ s,   _____ m/s

$t_f, v_f =$ _____ s,   _____ m/s

$\Delta v = v_f - v_o =$ _____ m/s

4.   Compare your two values for Δv percentage difference.

Percentage difference = _____ %

Show calculations here.

II. FREEFALL

With our more modern equipment we can find the acceleration due to gravity, g, without using the ramp to slow down the motion. And we can observe some interesting symmetries found when objects move with constant acceleration. We'll be leaving the Dynamics Track now to work with a video of a billiard ball tossed upward. We'll use a software tool that we call the Frame Flipper. The coding for this tool was written in 1996 by Joe Pearson. He went on to become Application Architect at The Weather Channel. Cool, I think.

In your browser navigate to the following location. **The QuickTime movies in the first two frames may take a while to load. It may help to click in each of these two frames.**

https://virtualphysicslabs.ket.org/wp-content/uploads/2018/12/Kine-acc/index.htm

A Dropped Object

In the first column you'll see a QuickTime movie of a dropped billiard ball which falls through a distance of about 2 meters. Click the Play button ▶ at the bottom of the movie. You can also double-click in the movie to start it. After playing the movie once you can also use your left and right keyboard arrows to step through the movie backwards and forwards.

1.   Briefly describe what you observe in this movie. Use physics lingo!

_____

_____

_____

An Object Thrown Upward

Now play the movie of the object thrown upward. This time you'll see the very smooth motion of a billiard ball thrown upward and then allowed to fall back down.

2.   How does the trip upward compare to the trip downward in this movie?

_____

_____

_____

3.   Notice how the **spacing** between the images and the **fuzziness** of the images varies as the ball rises and falls. What do these **two** observations indicate about the ball's motion? (Two answers.)

_____

_____

Hopefully you noticed that this **second, up and down motion**, is just like our ramp investigation. There's one minor but important difference. With our ramp, the ball moved first in a negative direction, then in a position direction. In this case the ball will travel in the positive, upward direction, then in the negative, downward direction. We're going to do a quick repeat of the ramp investigation, this time with the billiard ball. Using what you've learned so far you should be able to draw predictions for all three graphs.

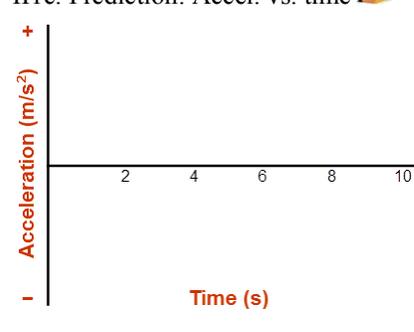4.   Sketch your graphs below. **Assume the upward direction is positive.**

II1a. Prediction: Position vs. time
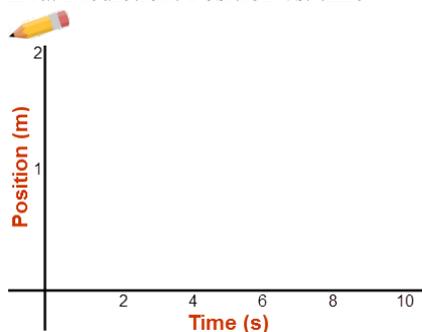
II1b. Prediction: Velocity vs. time
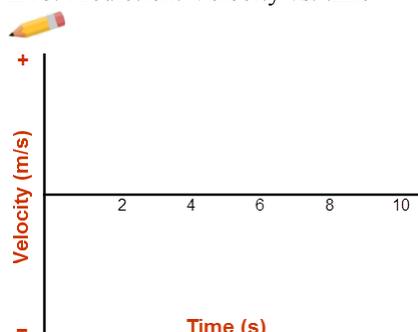
II1c. Prediction: Accel. vs. time

5.   What will be the sign of the acceleration in this movie? (+ or -) _____

OK, let's try the actual Frame Flipper (FF). Click the "Frame Flipper" link next to the movies to start it. It will open up a new window. You'll recognize the image as the first frame of the second movie. In this frame, the ball has just been set into motion. That is, it has just been released and is moving only under the influence of gravity and a negligible amount of air resistance. FF allows you to take position vs. time data of a moving object by clicking on it at each successive position. We'll be interested in the vertical position only, so ignore the x values.

If you run into any problems, you'll need to close the Frame Flipper and restart it.

Here are the steps.

In order to get actual measurements in meters we need to determine the scale of the image, that is, determine how many pixels tall the meter stick in the figure is.

        Do Steps 1 and 2.

We then need to set the y = 0.0 m point.

        Do Step 3.

Then we can start taking our data.

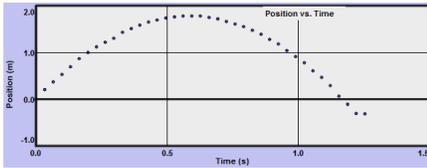   Do Step 4 which repeats until you've taken 38 readings.

You should be able to work this out by reading the directions on the screen. FF will automatically advance from Step 1 to Step 2 to Step 3 to Step 4 when you click on the screen. You'll repeat Step 4 for as many frames as the ball takes to rise and fall, (38), clicking the forward arrow each time to advance the movie. If you make an error, you can close the window and re-open it. Go ahead and collect all your data. It's a bit blurry, so take care. And near the top there won't be much motion. But you should expect that.

To copy and paste your t and y data from FF to Grapher, **you just need to copy the y-column**. The time data will come along with it. In FF, click anywhere in the y(m)-column. Right-click and choose "Select All", then right-click and select "Copy". Open Grapher and Paste (Ctrl + V) your data into the table. Scale your graphs as needed. You should have a nice down-opening parabola and its associated v-t and a-t graphs. (Hopefully that's what you predicted.)
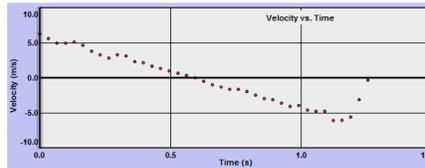
6.   Draw the three graphs produced by Grapher below.
      Note: II2c is very wiggly due to imprecision of frame-flipper data. You might also note that a small oops in the x-t graph will produce a couple of bad velocity points and about twice as many bad a-t points. This is a nice visual representation of what's called propagation of error. An error in one point in the y-t graph affects the calculation of the velocity in the two time intervals either side of it. Similarly the problem is compounded with the acceleration calculations.
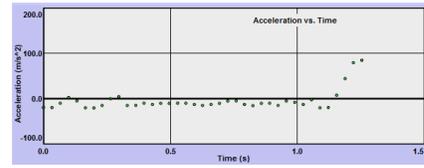
II2a. Observation: Position vs. time	II2b. Observation: Velocity vs. time	II2c. Observation: Accel. vs. time

7. From the movie you can see that the ball is at rest ($v = 0$) at the top of its motion. How do the *y-t* and *v-t* graphs indicate this?

_____

_____

_____

8. Does this mean that its acceleration is zero there? How do the *v-t* and *a-t* (sort of) graphs answer this?

_____

_____

_____

What we've found is that, **in the absence of friction, vertical motion is motion at a constant acceleration**. Note that the term vertical motion applies to either upward or downward motion. The magnitude of the **acceleration due to gravity, g** varies from location to location, but a typical value is **9.80 m/s²**. Its direction is always downward.

9. You can probably get your best experimental value for this acceleration from the slope of a well-behaved section of your *v-t* graph. Record this below.

Experimental value for g from *v-t* graph = _____ m/s²

10. Compare your experimental value to the accepted value by finding the percentage error between your experimental value and our accepted value.

Percentage error = _____ %

Show calculations here.

III. OBSERVATIONS OF SYMMETRY

We noted earlier that the *x-t* graph was symmetrical. But we've also stated that these graphs model the motions we observe. What symmetries should we find in our models and our motions?
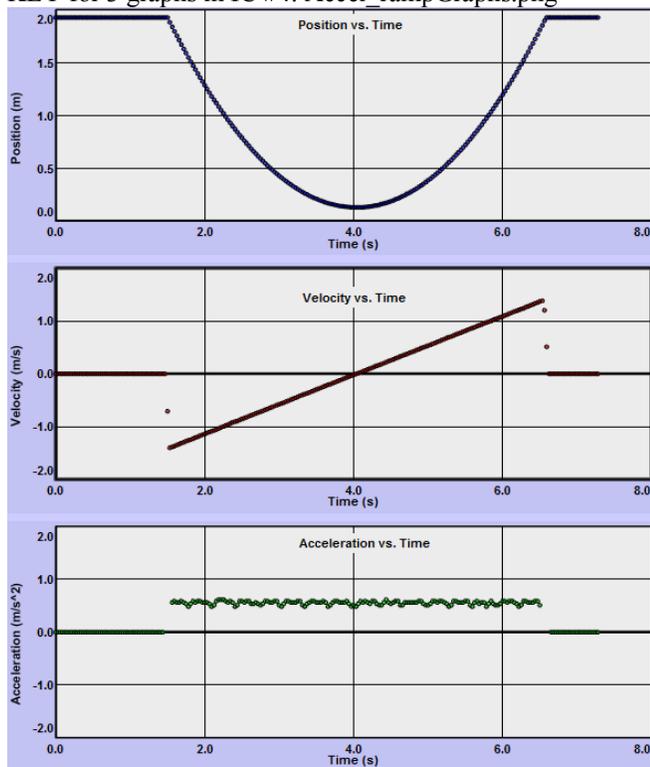
1. How does an object's acceleration while rising compare to its acceleration while falling? Explain your answer using either the *v-t* graph or the *a-t* graph.

_____

_____

_____

With this information and our five kinematics equations, we can solve problems with vertical motion. There are a couple of useful observations we can make from our data that will help with problem solving.

2. Using Grapher turn Examine and Tangent. As you move your pointer across the *x-t* graph a short tangent line will follow your cursor and the tangent at the nearest data point will display in the tangent text box. In the Examine box you'll see (time, position) values constantly updated. For some position, y on the left (ball rising) side of the graph, note the slope. Now repeat for the same position, y on the right (ball falling) side of the graph. (You won't be able to get identical heights. Just get close.) The magnitude of the slope is the speed. How do speeds compare at corresponding heights? Repeat for another pair of matching heights. State your observations.

_____

_____

_____

3. As you move your pointer across the position, time graph, the time to reach each y-position found in your data table appears in the Examine box.

Using this tool, find the time the ball took to rise between a point near y = .5m and y = 1.5m.   _____ s

Find the approximate time between the same positions on the way back down.   _____ s

Pick another similar pair and repeat. (Again, this will just be approximate.)

4. State your observations.

_____

_____

_____

KEY for 3 graphs in IC #4: Accel_rampGraphs.png

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