Name __________________________________ School _____________________________________ Date ____________

Dynamics – Pulleys, Ramps, and Friction

PURPOSE

- To investigate the vector nature of forces.
- To practice the use free-body diagrams (FBDs).
- To learn to apply Newton's Second Law to systems of masses connected by pulleys.
- To develop a clear concept of the idea of apparent weight .

EQUIPMENT

Virtual Dynamics Track VPL Grapher PENCIL

EXPLORE THE APPARATUS

Open the Virtual Dynamics Track. You'll see the low-friction track and cart at the top of the screen. At the bottom you'll see a roll of ***massless*** string, several masses, and a mass hanger. Roll your pointer over each of these to view the behavior of each. Note the values of the masses. Also note that the empty hanger's mass is 50 g. If you move your pointer over the cart you'll see that its mass is 250 g.

- Drag the cart near the left end of the track.
- Drag the spool of ***massless*** red string somewhere just to the right of the cart. (Fig. 1a.) Release it by releasing your mouse button. A segment of string will attach to the cart and find its way over the pulley. A small loop (Fig. 1b.) will form at its lower end.
- Drag the mass hanger until its curved handle is a bit above the loop (Fig. 1c). Release your mouse button and the hanger will attach. (Fig. 1d) The cart will take off. It's alive! Drag the cart back and forth. Everything should work just as you'd expect. Give it a toss or use Go→ to launch it with a known initial negative velocity.
- Check out the Brakes On/Off toggle button.
- Try Reset All.
- Reattach the string and hanger.

Turn on the brake and move the cart near the middle of the track. (Just so you can see the hanger.) You can add to, and remove masses from, the cart or hanger as needed. Drag the largest mass, 200 grams, and drop it when it's somewhat above the base of the spindle of the cart. (Fig. 1e) Drag a 50-g mass onto the hanger.

We refer to this group – cart, string, and hangers – as a **system**. You can actually call any group of objects a system – the solar system for example. We're going to investigate how our cart, string, and hanger move together as a system. The massless string transmits forces between the cart and hanger.

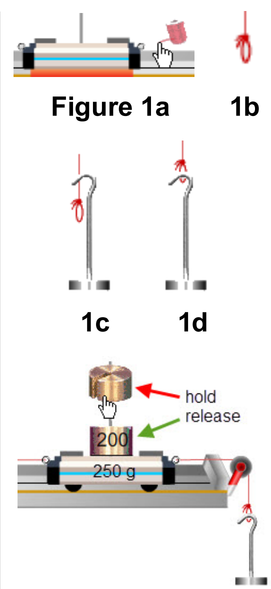


Figure 1 – Adding Strings, Hangers and Masses

- Toss the cart around and notice how it moves. Experiment with a variety of masses on both the cart and hanger. Investigate what determines the system's acceleration. You know how to *add* masses. You can *remove* them just by clicking. Try this. Turn on the brakes to hold the cart still and then add two masses to the cart and to each hanger. Click once on each of the three. The top mass will disappear in each case. Another three clicks will empty them all. Clicking on an empty hanger will remove it. The Reset All button will then remove the massless string.

We'll now use this and other arrangements of masses to look at various systems, how they move, and why. We'll quantify what we find using Newton's second law, $\Sigma\vec{F} = m\vec{a}$, where **$\Sigma\vec{F}$ is the net, external force on the system**, **m is the total mass of the system, and $\vec{a}$ is the acceleration of the system**.

PROCEDURE

I. The Vector Nature of Forces; Free-body diagrams (FBDs)

We'll start with a puzzle. This will help focus your attention on what can be accomplished by the creative use of Newton's Second Law. This is also the most complex part of the lab. (You're welcome.)

With the brake off, 200 g on the cart for a total of 450 g, and the empty mass hanger (50 g) you should see that the cart wants to stay against the right bumper. We now want to investigate the forces acting on this system when we tilt the ramp.

- Click on one of the small rectangles on either end of the track, near the pulleys. (See Fig. 2.) You can now drag up or down to adjust the track angle through a range of ±12°. The cart will respond just like you'd expect.
- Try adjusting the ramp to a specific angle like 5.7°. It's a bit hard to be that precise. Try this. Click on the left end of the track and with the mouse button still down move your pointer over near the middle of the track. You can still adjust the angle by dragging up and down, but not very precisely. Now, with the mouse still down, drag way out to the left past, past the sensor. You can still adjust the track angle, but with much greater precision.
- Now, using this new skill, and with the same masses try to find an angle where the cart will remain motionless. Note that you have to over-tilt to slow it down, and then reduce the tilt when it's about stopped. You can also play with the brake.
- Give up? You can't exactly make it stop with these specific masses since the angle is only adjustable to within .1°. But you can get close using the brake to calm things down.

  Turn on the brake and set the track angle to 6.4°. Drag the cart to the middle and release. It just barely accelerates down the track. So 6.4° is a bit steep. Now try 6.3°. Now it just barely accelerates up the track.
  So we can't find the angle experimentally, but with our recently-developed mathematical skills we should be able to find just what the balancing angle is.

1. Before we get to the math, state a <u>possible</u> value for the balancing angle. θ = ______________ °

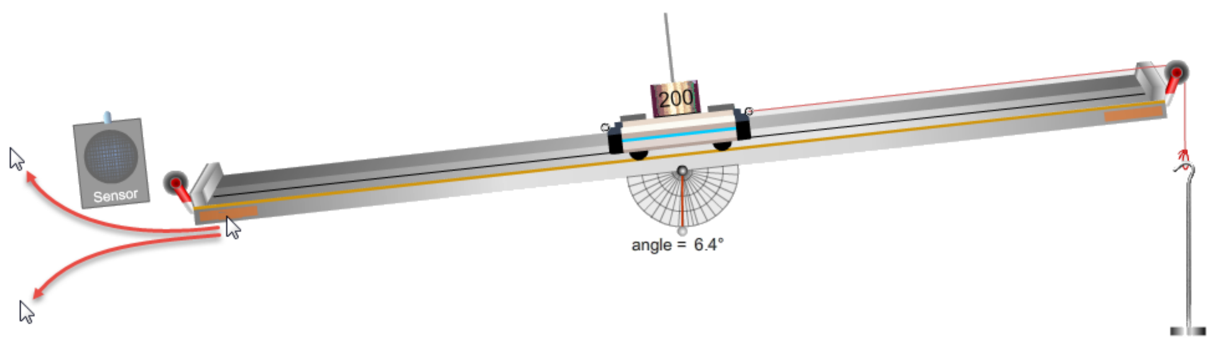


Figure 2 – Adjusting the Track Angle (The vertical section of string has been shortened to reduce the size of the figure.)

It's going to get pretty intense now. We'll first have a thorough look at the force and motion vectors for the hanger and cart. It will become less abstract once you start to look at a few scenarios. The Dynamic Vectors are your most important ally. They're explained in part B. It's recommended that return to them frequently.

A. Forces on the Mass Hanger

In Figure 3 we illustrate the forces acting on our mass hanger. In Figure 3a we arbitrarily pick the +x direction for the hanger as downward. (This will seem more reasonable when we connect it to the cart.) Then in Figure 3b we see a mass hanger free-falling downward at g as it would if there were no string attached. The net force is due entirely to the force of gravity on the hanger, the hanger's weight. The resulting acceleration is "g" as expected. Make sure you understand the math in each figure. These forces and accelerations are all vectors. And we're using vector addition. That's where the signs come from.

You can actually see this freefall by clicking on the Zombie Cart icon.  It and the actual cart will become translucent zombie carts to indicate that the cart has become massless. After you try this, click the icon again to bring the cart back to life.

In Figure 3c a string with upward tension T will change the net force and as a result, change a_x. There are three possible resulting accelerations. (You'll understand why we were so careful with the sign of the acceleration in the acceleration lab.)

- If $T > W$, the net force and acceleration will be upward (negative).
- If $T < W$, the net force and acceleration will be downward (positive).
- If $T = W$, the net force and acceleration will be zero, thus maintaining its (constant) velocity or state of rest.

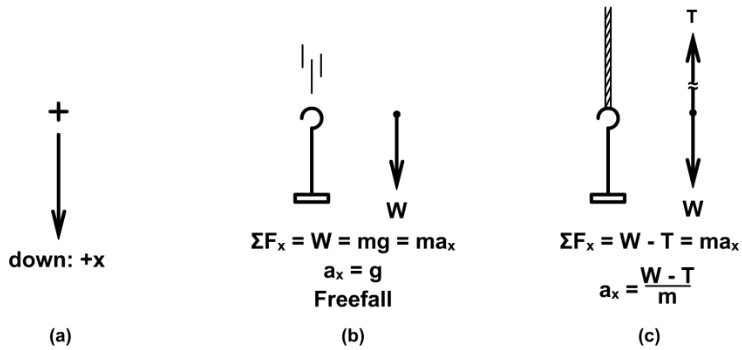


Figure 3: Forces on the Hanger; FBDs

B. Forces on the Cart

The forces on the cart are more complex since the downward force of gravity acts at an angle to the track thus acting somewhat down the ramp and somewhat into the track. For this reason we need to consider forces both parallel to the ramp and perpendicular to it. **The parallel forces determine the acceleration of the cart along the track, while the perpendicular forces determine the amount of friction between the cart and the track.**

Keep the masses on the cart and hanger as they were in Section 1, but set the track angle to 0°. Turn on the brake and move the cart over near the left end of the track. In Figure 4a you see the three forces acting on the cart on a level track assuming no friction. (There's an extra copy of Figure 4 at the end of the lab for your convenience.)

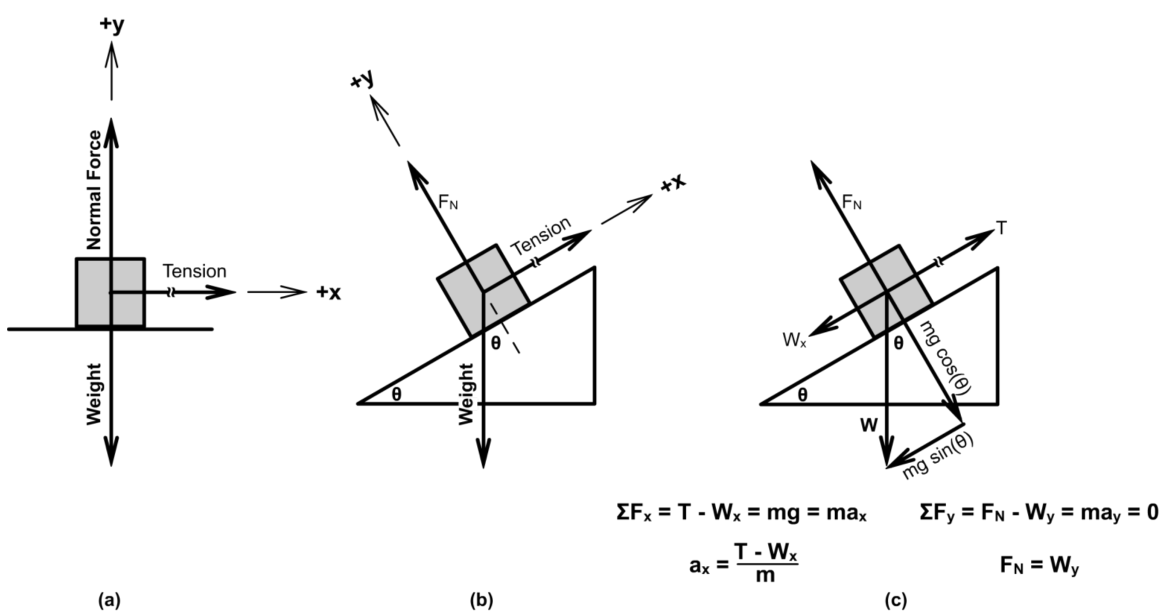


Figure 4: Force on the Cart; FBDs

To see a real time representation of all these force vectors, turn on Dynamic Vectors by clicking on the icon. At the center of the track you see a collection of vectors of various colors. Some of them have zero magnitude but their names are still displayed. On the left side of the screen there is a legend explaining the colors and lengths of the vectors.

Drag the cart from side to side to see the velocity vector change in magnitude and direction. Also note that Figure 3c is displayed above the weight hanger. (r: right side) Turn the brake off and on to see everything changing with the situation.

Toss the cart around. Tilt the track up and down to see all the components.

Now back to our investigation. Reset θ to 0°. Turn on the brake and move the cart away from the center. The cart's weight acts downward, currently perpendicular to the track. The resulting equal magnitude normal force acts upward.

We pick the +y direction to be perpendicular to the ramp and upward. This is convenient since the normal force always acts in that direction.

We pick the +x direction for the cart parallel to the ramp and to the right. This is also a convenient choice of axis since the tension will be along that axis as will the cart's motion.

When we tilt our ramp (try it) at an angle θ (Figure 4b), the x and y-axes also rotate through the angle θ and we now have an angle θ between the y-axis and the weight vector which is still downward. The x-axis and tension remain parallel to the ramp.

In Figure 4c all of the forces and components of interest are shown. The weight vector is resolved into its x and y components. Note that in the lab apparatus the orange Normal, and the light pink W, W_x, and W_y vectors are drawn half-size. There's a darker pink second W(x) on the track which is drawn full size.

- The weight vector's x-component, W_x, equals $mg\sin(\theta)$ and represents the extent to which the weight vector acts parallel to the ramp. It acts in the –x direction for positive track angles.

  Opposing W_x is the tension force acting in the +x direction. (Plus F(f) from the brake if it's on.)

  Any friction forces would also act along this axis. Their (±) direction depends on the situation. (This can be a major source of difficulty for most students. Be sure to revisit this apparatus for clarification when you need it.)
- The weight vector's y-component, W_y, equals mg cos(θ) and represents the extent to which the weight vector acts perpendicular to the ramp. It acts in the –y direction. As we've observed, our cart doesn't accelerate along the y-axis so it must have an equal and opposite force acting on it in the +y direction. This is the normal force, F_N acting on the cart. It determines the amount of friction acting. ($F_f \leq \mu F_N$ depending on what other forces are present.)

Similar to the mass hanger, the cart's motion is determined by the relative values of T and W_x. There are three possible resulting accelerations. (Careful! Accelerations, not velocities.) Again, use the dynamic vectors to clarify this. With the brake off, observe the W_x, T_r, and F_{net} vectors at about 5° and at 8°. Throw the cart up or down the ramp as needed. Watch F_{net} as you adjust the ramp angle up and down between those angles. You should observe the following:

- If $T > W_x$, the net force and acceleration will be up the ramp (positive).
- If $T < W_x$, the net force and acceleration will be down the ramp (negative).
- If $T = W_x$, the net force and acceleration will be zero, thus maintaining a constant velocity or state of rest.

Before you continue, be sure to verify that the velocity's direction is immaterial. For each case throw the cart and watch the velocity change direction as the car recoils off the ends. Note that the other vectors are unaffected. (You can't really verify this for the third case since we've seen that we can't hold the cart still by adjusting the angle, but you should get the idea.)

C. Forces on our two-mass system

Where were we? Oh yeah, we said

> "So we can't find the angle (where the cart will remain at rest) experimentally, but with our recently-developed mathematical skills we should be able to find just what the balancing angle is."

Because our two-mass system is joined by a taught, massless string, the whole system moves with a common acceleration. The direction of this acceleration is different for each mass because the pulley redirects the tension force in the string. We've eliminated that problem by our selection of an x-axis "with a bend in it."

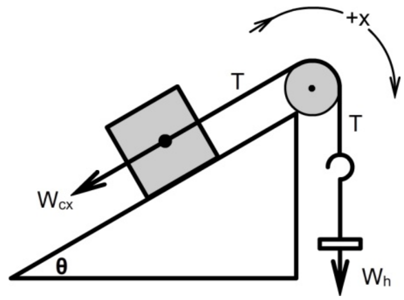


Figure 5 – The +x Direction for Our Ramp System.

If you step back and have a look at the **system** of one cart, one hanger, and a connecting string, you can see that the forces that actually accelerate the system, that is the external forces, are the weight of the hanger W_h, and the x-component of the weight of the cart, W_{cx}. The tension, T, is actually an **interior force** within the system. We can write the second law for this two-mass system by combining equations we wrote for each part of the system. Note that the tension force is an external force for each of the individual parts, but an internal force to the whole system. Writing $\sum \vec{F}_x = m\vec{a}_x$ for each part of the system we find:

Mass Hanger: $W_h - T = m_h\, a_x$

Cart: $T - W_{cx} = m_c\, a_x$

We can eliminate the tension by adding our two equations. We'll add the left sides together and the right sides together.

For the two-mass system we have $\boldsymbol{W_h - W_{cx} = (m_h + m_c)\, a_x}$ (1)

Equation 1 looks just right. It says that the net external force equals the total mass of the system times its acceleration.

In our particular quest to find the angle where the system will sit still or move at a constant speed, we can go one step further since for our **equilibrium situation,** $a_x = 0$. Thus,

$$W_h - W_{cx} = (m_h + m_c)\, a_x = 0$$

so $W_h = W_{cx}$

From our diagrams we know that for the hanger $W_h = m_h\, g$

And for the cart $W_{cx} = m_c\, g \sin(\theta)$

So, equating W_h and W_{cx}, $\boldsymbol{m_h\, g = m_c\, g \sin(\theta)}$

1. Let's try it. With a 50-g hanger and a 450-g cart, the angle for equilibrium, θ = ____________ °

Show calculations here.

So now we see why our cart accelerated in one direction at 6.3° and in the other at 6.4°. This $\Sigma\vec{F} = m\vec{a}$ equation is a pretty powerful tool.

2. Here's an interesting follow-up question to debate with your lab partner. If you added a further 50 grams to each member of the system – the cart and the hanger – would the equilibrium angle (when a = 0) increase or decrease? Explain your reasoning. Think about the relative effect of adding 50 grams to each mass.

____

____

____

3. Calculate that new equilibrium angle for this arrangement.______________ °

Show calculations here.

4. Try it. If your results don't match your prediction, maybe you need to check your figures.

II. Accelerated System of a Cart on a Level Track with Mass Hanger and No Friction

In this part of the lab you'll be working with a much simpler arrangement - a level track with no friction. It would probably help to leave the dynamic vectors on. By now you should need less explanation of how the apparatus works as well as how we develop the mathematics.

1. Set up your system with 200 grams on the cart and an empty mass hanger on the right side. (Figure 6.)

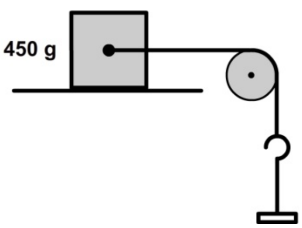


Figure 6 – Cart and Hanger; Horizontal Track

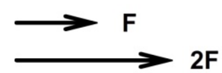


Figure 7 – Force Vectors Drawn to Scale

Important: In all the vector diagrams you'll draw, use a consistent scale for the force vectors as in Figure 7.

In Figure 8a you see a drawing of a mass hanger in free fall along with a corresponding free-body diagram.

In Figure 8b draw a drawing and an FBD for your mass hanger when it's connected to the cart and the brake is on. You'll just draw 1) the cord and mass hanger, and 2) T and W_h vectors added in an FBD. **Look back at earlier figures and at the Dynamic Vectors on your lab apparatus for help. The apparatus will show you all these vectors.**

8a. Drawing and FBD for **hanger** in **free fall**.	8b. Drawing and FBD for **hanger** with **brake on**.	8c. FBD for **hanger** with **brake off**.

Now, turn off the brake and observe what happens to the system (cart and hanger)? Big surprise, huh? Clearly Figure 8b no longer applies. The hanger is falling so… for Figure 8c draw **just the FBD** for the hanger in this situation. This may call for another debate with your lab partner. Be sure to turn the brake on and off and watch the weight and tension vectors as the cart stops and moves. But your drawing for 8c is for the brake off situation. That is, when the cart is moving.

2. Let's now do the same for the cart. Remember to keep the same scale as in #1.

In Figure 9b draw the FBD for the cart with the hanger attached and the brake on. Use F_B for the brake's force and T for tension.

In Figure 9c draw the FBD for the cart with the hanger attached and the brake off.

	9b. FBD for **cart** with mass hanger attached and **brake on**. (just horizontal forces)	9c. FBD for **cart** with mass hanger attached and **brake off**. (just horizontal forces)	

We now have a pair for FBDs (8c and 9c) for our system when the brake is off and it's allowed to accelerate. Here's a figure which includes our redirected x-axis and the forces along that axis. Figures 8c and 9c should include only these two forces.

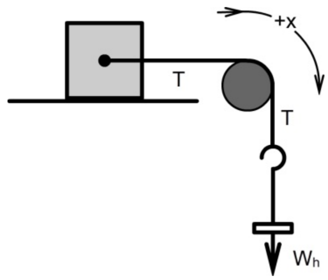


Figure 10 – Forces Acting on a Cart and One Hanger along x-axis

3. Your goal now is to determine the acceleration of the system and the tension in the string. In Section 1 of this lab we worked with a similar, but more complex system which was in equilibrium. Feel free to use it as a guide. Remember, this time we're not in equilibrium. This time there is an acceleration. Here are a few pointers.
   - In Section IC we used FBDs for the cart and hanger and Newton's Second Law to create two equations, one each for the mass hanger and the cart. From these we produced Equation 1 which describes the acceleration of the system in terms of the masses and forces acting along the direction of motion. A similar, simpler equation can be derived for this system.
   - Each FBD (8c and 9c) is a blueprint for constructing a statement of Newton's 2nd Law. Start by writing these equations down. Be sure to include subscripts (h for hanger and c for cart) for all mass and weight terms
   - Solve these equations simultaneously to find an equation for the full system just as we did in Section 1C. Use it to calculate the acceleration of the system.

     Your answer should look familiar. After all you're exerting a net force on a system equal one tenth of its weight.

Show calculations here.

a = ___________ m/s^2 T = ___________ N

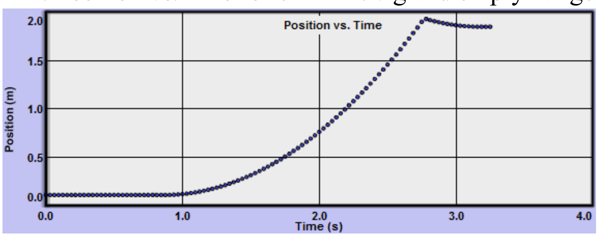
Confirm your results experimentally.

Using your virtual apparatus, confirm your results. If you recall, in lab 2.3 you found the acceleration of a cart from the slope of a velocity, time graph. That would be a good method to use here. Look back to that lab for guidance.

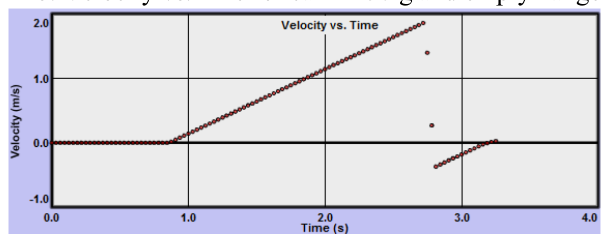
4. Sketch the graphs you obtain from **Grapher**. (Turn on all three graphs to reduce the size of the graphs.)

5. Take Screenshots of your *x-t* and *v-t* graphs. Save them as "Dyn_IIa.png" and "Dyn_IIb.png", print them out, and paste them in the space provided.

11a. Position vs. time for cart + 250 g and empty hanger



11b. Velocity vs. time for cart +250 g and empty hanger



6. Acceleration determined graphically__________m/s^2

7. Compare the theoretical (step 3) and experimental (step 6) values for this acceleration.

Percentage error _____________%

Show calculations for percentage error here.

III. Accelerated System of a Cart on a Level Track with Two Mass Hangers, No Friction

It's a simple extension to add a mass on the left-hand side. Let's look at it briefly. In Figure 12 we've added a second mass hanger. As a result we've had to add more subscripts to distinguish the hangers and tensions on the left and right. Note also that the positive direction is "up" on the left, "right" on the track and "down" on the right.

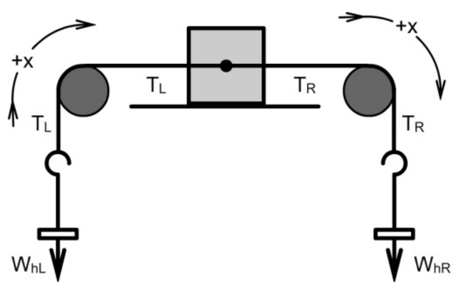


Figure 12 – Forces Acting on a Cart and Two Hangers along x-axis

Suppose both hangers are empty and the cart has the usual extra 200 grams on it. So

$m_{hL} = 50$ g $\quad m_{hR} = 50$ g $\quad m_c = 450$ g

1. Is this system in equilibrium? Test it with your apparatus. Yes No (Circle one.)

2. What are the magnitudes of the two tensions? T_L =_______ N T_R =________ N

Now, suppose we added 20 grams to the right hanger. The system will certainly accelerate in the +x-direction. Go ahead and try it with your apparatus just to make sure.

3. How do T_L and T_R compare now, while the system is accelerating?

$T_L > T_R$ $\quad T_L < T_R$ $\quad T_L = T_R$ (Circle one.)

Puzzled? There's more than one way to think about this. Here's one way. We know that

$T_L > .49$ N since it's able to accelerate the left hanger upward. And

$T_R < .69$ N otherwise the right hanger would not accelerate downward.

But that's not enough to answer the previous question. In Section II you calculated the acceleration and tension after deriving equations from Newton's 2nd law. Maybe we can find the tensions that way. This time we have three masses so we'll have a 2nd law equation for each of the three masses.

4. Draw the three FBDs for this situation. Be sure to include the subscripts and **use a consistent scale**. You should find that each of the four forces (W_{hL}, T_L, T_R, and W_{fR}) has a different magnitude, so you'll need four different lengths.

13a. FBD for left hanger	13b. FBD for cart (just horizontal)	13c. FBD for right hanger

From that we can create our three equations.

1) $T_L - W_{hL} = m_{hL}\, a$ left hanger

2) $T_R - T_L = m_c\, a$ cart

3) $W_{hR} - T_R = m_{hR}\, a$ right hanger

Assuming that we know the masses, this gives us three unknowns – a, T_L and T_R. That shouldn't be hard to solve. Adding the three equations should eliminate the internal tension forces.

$$(T_L - W_{hL}) + (T_R - T_L) + (W_{hR} - T_R) = m_{hL}\, a + m_c\, a + m_{hR}\, a$$

$$W_{hR} - W_{hL} = (m_{hL} + m_c + m_{hR})\, a \qquad (2)$$

Make sense? All this says is that the net force on the system is the vector sum of the two external forces and that the mass being accelerated is the total mass. You can go straight to this final equation once you catch on the whole system idea. $\Sigma F = ma$ refers to the external force on and mass of any group of objects.

5. Find the **acceleration** of this three-mass system and the **two tensions**.

Show calculations here.

a = ___________ m/s^2 T_L = ___________ N T_R = ___________ N

We predicted that $T_L > .49$ N and $T_R < .98$ N . Hopefully that checks out ok.

IV. Atwood's Machine

Atwood's Machine, shown in Figure 14a, is similar to our previous system but simpler. Without the mass in between the two pulleys, the tension is the same throughout the string. So we just represent it everywhere as *T*.

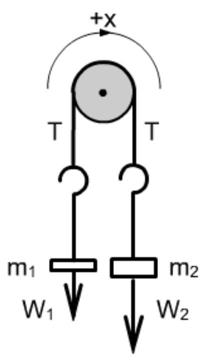


Figure 14a. Atwood's Machine

1. With the cart and two hanger system we just worked with, F = ma became

$W_{hR} - W_{hL} = (m_{hL} + m_c + m_{hR})\ a$ Equation 2

Atwood's machine essentially just removes the cart. Write down $F = ma$ for each of the two masses and then combine them to find an equation similar to Equation 2. Show your algebra here. Use the mass and weight notation from Figure 14a.

Atwood's Machine Equation: ____________________________ (3)

This would be a good time to note that we are using ***massless*** string, which means that its density is so small that it can be neglected in our analysis. How would you deal with the weight and mass of real, non-massless string as it moved from one side to the other? You'll need to go beyond algebra to solve that one.

OK, let's try it in the lab. Oops! We have a problem. All our data collection tools depend on the presence of the cart. So we need the cart to be present but we don't want its mass to be present. This calls for Zombie Cart!!!!!

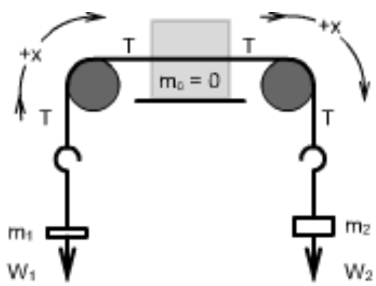


Figure 14b. Our Version of Atwood's Machine.

1. Start with system shown in Figure 12. Use m_L = 50 g, m_R = 70g. If you then click the Zombie Cart button you'll just have the two hanging masses. There are two pulleys, but there is nothing in between (except the soulless/massless zombie cart) so it's equivalent to the Atwood's machine in Figure 14a. Having a second pulley and the string in between adds nothing to the system. It's just two masses connect by a string with a common tension.

   With the real cart present there was a different tension on each side of the cart. Notice how the tension is the same everywhere in Figure 14b. If it weren't you'd have a tension that changed along the horizontal string. That would be hard to explain, unless you had a string with mass, which we don't.

2. Calculate the theoretical acceleration using Equation 3.

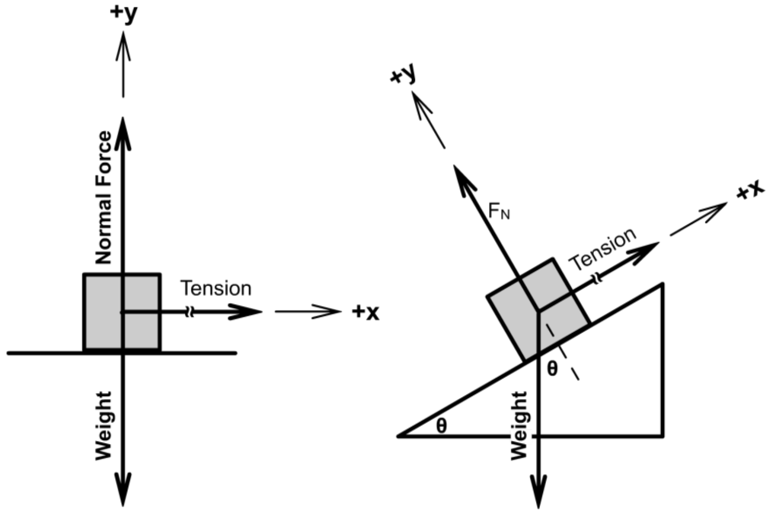
Show calculations here.

3. Get things set up to take data, start the sensor and then click the zombie button.

   Experimental acceleration from Grapher _____________ m/s^2

4. Determine the percentage difference between your experimental and theoretical values for the acceleration.

Show calculations here.



$$\Sigma F_x = T - W_x = mg = ma_x \qquad \Sigma F_y = F_N - W_y = ma_y = 0$$

$$a_x = \frac{T - W_x}{m} \qquad F_N = W_y$$

(a) (b) (c)

Figure 4: Force on the Cart; FBDs

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